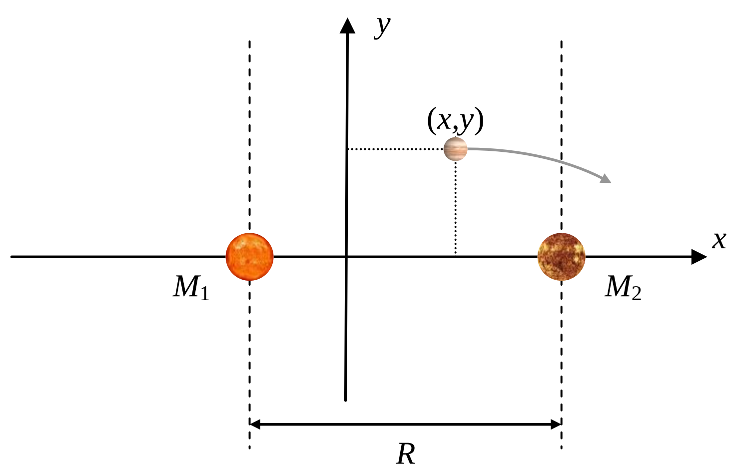
Planets around binary stars – an investigation of orbits and temperatures

Introduction

In our solar system, each planet has a theoretical equilibrium surface temperature that ensures that the power it radiates balances the sum of the power it receives from the Sun and the power it generates internally. Since the latter is reasonably steady, and the orbits of the planets are not very eccentric, this temperature does not change much as the planet circulates about its orbit.

To make things more interesting, we consider in this practical, the orbits of planets in a binary star system. In principle there are lots of possible systems and orbits to investigate, so to simplify things we restrict ourselves to considering a planet that orbits in the orbital plane of the binary. In addition, we will have the two stars in the binary orbiting their common centre of mass in circular orbits. This set-up has the nice feature that the separation of the two stars remains fixed and we can move into a rotating frame of reference in which they are stationary.



The Program

Open **binary.xls**. The program plots orbits in the rotating frame and also plots the temperature on the planetary surface. To run the program, you press **ORBIT** and input the initial conditions of the orbit and the nature of the planet in the GUI.

The program will place the origin of coordinates at the centre of mass for any choice of star masses.

Let's take a look at the parameters.

Mass of star: self-explanatory. If you set one of the masses as tiny and make the separation reasonably large, you will recover our solar system. Remember for a binary system the angular velocity Ω is given by $\Omega^2 = \frac{G(M_1 + M_2)}{a^3}$. In this program, the planets are orbiting in a **clockwise sense**.

Separation: This is the centre-to-centre separation of the two stars.

Luminosity of star: Remember the solar luminosity is 3.87 in these units.

Run time: This is the length of time for which the program computes the orbit. Now you may be tempted to turn this up to very high values. However, the program has a fixed upper limit of time steps so that it does not become unreasonably slow to compute the orbit. If you make the run time too large and have insufficient time steps, the orbit will not be calculated accurately. You will notice a small message box appears after a run to tell you that the orbit is trustable (accurate integration) or potentially unphysical (inaccurate integration). If you get the latter, try increasing the number of time steps in the **Orbital Parameters** box and re-computing the orbit.

Position: All displacements on the plots are in AUs.

Velocity: For guidance, the Earth orbits the Sun with a circular speed of 29 kms^{-1}. However, remember, we are in a rotating frame (that is spinning clockwise in physical space).

Albedo: The fraction of incident radiation reflected by the body

Intrinsic temperature: If the planet has an internal energy source it will have a surface temperature even if not irradiated by a star. The default value corresponds to the value of temperature that Jupiter would have as a free-floating planet.

Procedure

Start with the default values and run the program. If the integration is inaccurate, increase the number of steps and run the program again. You will see that two plots are created: one is a plot of the planetary orbit in the plane of the binary; the other is the surface temperature of the planet as a function of time. The program makes a simplifying assumption that, even where the orbit is changing rapidly, the surface comes into thermodynamic equilibrium. You can see this may not be the case by comparing the temperature you achieve by standing adjacent to a radiator or brushing past it.

• From the temperature graph, locate the closest approach the planet makes to each of the stars. Make a note of the temperature and calculate the distance of the planet from the star. Verify that your result is consistent with the formulae you have met in the lectures for the surface temperature of a planet (see below).

Refresher on Surface Temperatures

The power absorbed by the planet is given by:

$$P_{abs} = (1-A)\pi R^2 \left(\frac{L}{4\pi r^2} \right).$$

The quantity $\frac{L}{4\pi r^2}$ represents the energy per unit area per unit time at the distance *r* form the Sun (or extrasolar star). Notice that the planet has an effective absorbing area identical to a disc of radius R (can you figure out why?). The quantity represented by *A* is the albedo. It represents the fraction of incident radiation reflected by the body - to give some examples, Venus has an albedo of 0.76, Earth 0.35 and Mars 0.16.

The radiated power can be estimated by assuming that the planet's surface emits like a black body. Remembering that the surface area of a sphere of radius *R* is $4\pi R^2$ and that a black body at temperature T emits energy per unit time per unit area proportional to the (large) fourth power of the temperature σT^4:

$$P_{rad} = 4\pi R^2 . \sigma T^4 .$$

Set the two expressions for power radiated and power absorbed equal to each other and you should find that the equilibrium temperature is;

$$T_{eq} = \left\{ {(1-A)L} \big/ {16\pi\sigma r^2} \right\}^{\frac{1}{4}} .$$

• Assess, at the two points you have found, how much of the surface temperature is due to the more distant star.

• Now run the same orbit with different values of the albedo and with the intrinsic temperature equal to 0 K. Write your results in your logbook and plot an appropriate graph to best exhibit the dependence of the surface temperature on the albedo.

• Describe, in a sentence or two, three different types of intrinsic energy in a planet that can lead to an intrinsic surface temperature.

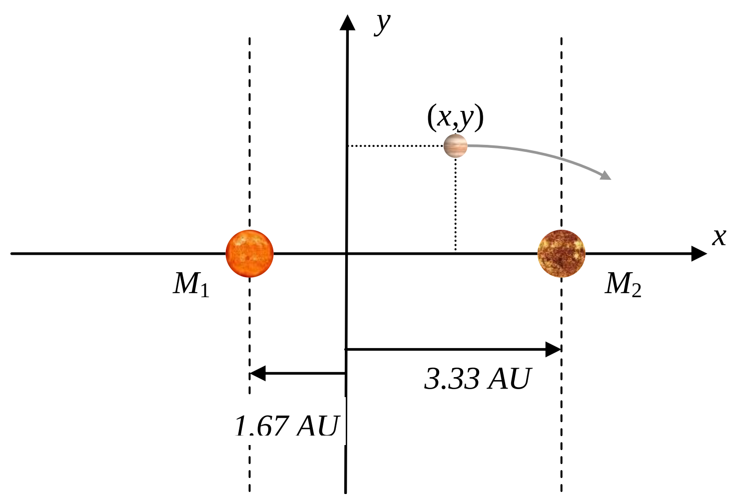
• Keeping the albedo fixed, repeat this procedure but change the luminosity of the two stars. For simplicity keep the masses fixed. However for any luminosity you choose find and record a plausible mass by doing some background research on the mass-luminosity relation for stars.

• Make very small changes to the velocity of the planet and keep the graphs that result from your experiments. It is best to separately change the X and Y velocities by very small amounts. Using your library of orbits, comment on the likelihood of finding an orbit that switches from one star to the other.

• Now find initial conditions to create three distinct orbits: one that orbits continuously around star 1; one that orbits continuously around star 2; one that encircles both stars. Collaborate with your friends to try different input values to help in the search. The section below gives some hints. (However use your own values for the two stellar masses and their separation).

Dynamics in a rotating frame – an example

Suppose our stars have masses of M_1 = 2 solar masses and M_2 = 1 solar mass respectively. Suppose they are separated by 5 AU. Then confirm the positions of the stars with respect to the centre of mass are correctly sketched below.



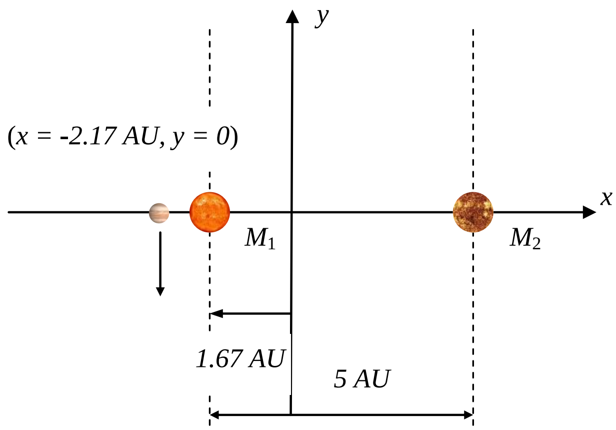
Then their orbital angular velocity around each other is $\Omega = \left(\frac{G(M_1 + M_2)}{a^3} \right)^{1/2}$ which equals 3.04×10^{-8} s^{-1}. So although the stars will appear stationary in the program, they are really moving with speeds $r\Omega$ with r = 1.67 AU for mass 1 and 3.33 AU for mass 2. Work out these two speeds: you should find 7.6 kms^{-1} and 15.2 kms^{-1}.

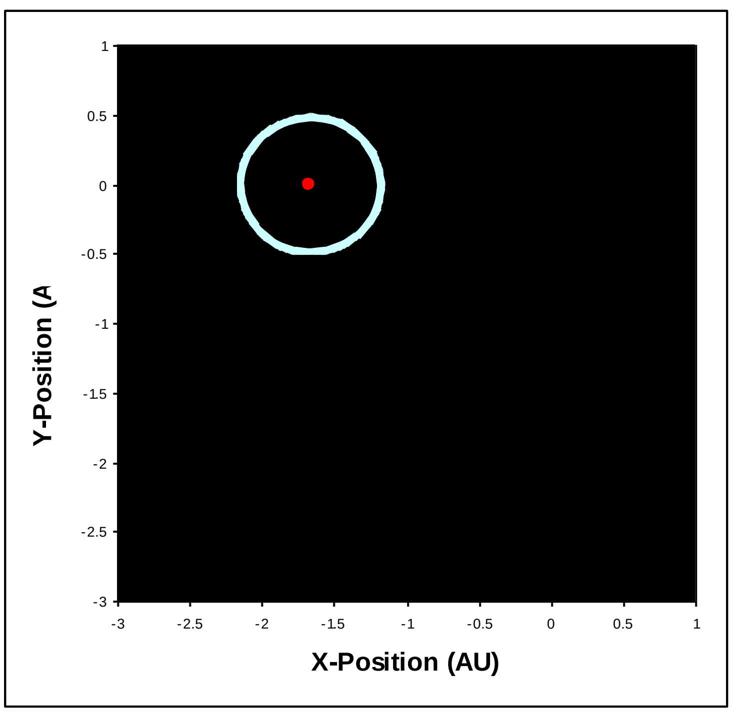
Now imagine a planet orbiting around star 1. It needs to be close to avoid a strong pull from star 2. Let's say it has an orbit with semi-major axis a = 0.5 AU. If the star was isolated in space the circular speed of the planet around the star would be

$a\Omega_2 = a\left(\frac{GM_2}{a^3}\right)^{1/2}$ =58.8 *kms*$^{-1}$. Suppose we place the planet in the initial position illustrated below. If we were in a stationary frame, it would need a velocity of (0, -58.8) to maintain a circular orbit.

However, the speed of the point (-2.12 AU, 0) in the rotating frame is 9.6 *kms*$^{-1}$(hint: 2.12 x 1.496 x 10^{11} x Ω). The speed of Star 1 is 7.6 *kms*$^{-1}$(1.67 x 1.496 x 10^{11} x Ω).

So because of the rotating frame, there is a 2 *kms*$^{-1}$ relative velocity offset of the planet relative to star 1. So we should use an initial velocity of (0, -58.8 -2) = (0, -60.8).



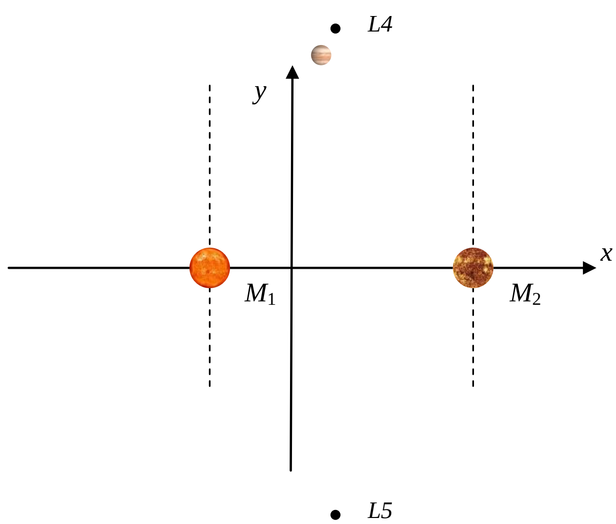


• The resulting orbit is shown above. You can see it's nearly but not exactly circular. There are (at least) two reasons for this – can you suggest what they are? From your graph, estimate the eccentricity of the orbit (i.e. using your mouse find the apastron (a(1+e)) and periastron distances (a(1- e)), and hence deduce the eccentricity).

Some projects

In the remaining time available to you, try one of the following investigations or come up with one of your own.

1. **Trojan Planets**: The L4 and L5 Lagrangian points in this system always form an equilateral triangle with the two masses as illustrated below. Investigate the stability of a planetary orbit started very close to (but not on) one of the Lagrangian points with zero velocity. Try different mass ratios for the two stars. You should find that when the mass ratio is large enough, the planet can orbit around the Lagrangian point. Try and find the critical value of this mass ratio where stability first becomes possible.



2. **Life around binary stars**: Investigate habitable planets in a system of your choice – take habitability to be the presence of liquid water and use this constraint as a constraint on surface temperature