Silent Witness – Using Physics and Mathematics to Decode Images

The aim of this masterclass is to show you how mathematics can be used to 'decode' images.
Some applications in the everyday world are:

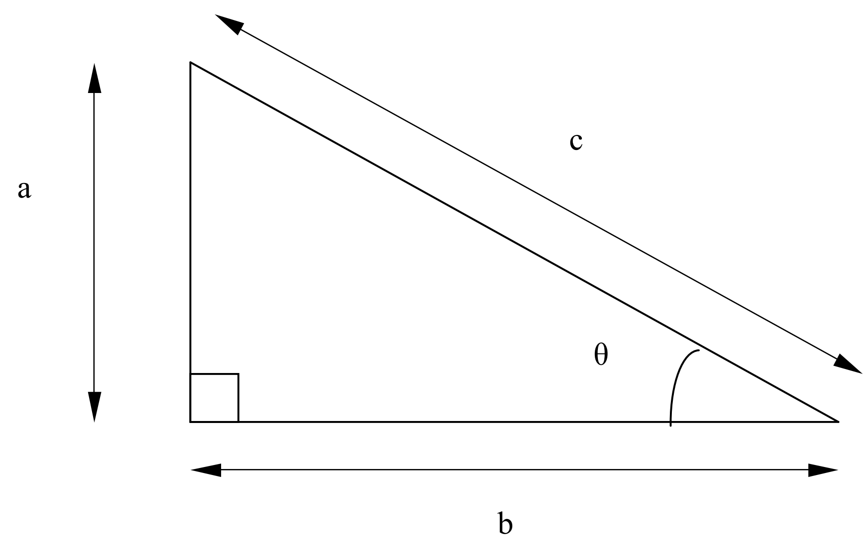
- forensic work;
- trying to recover information from blurred images e.g. from security cameras;
- understanding images taken from spacecraft or spy satellites;
- movie making particularly computer-generated imagery (CGI).

Although these are all very contemporary topics, the decoding of images has a long tradition – we'll see some examples at the start of the session.

In this Masterclass, you will make a lot of use of trigonometry. You may not have met all of the tools we will use before – don't worry, the best way to learn something new is to use it.

Here are some useful bits of trig that will come in handy.

Pythagoras' theorem
This relates the lengths of the three sides of a right-angled triangle: $a^2 + b^2 = c^2$



Sine, Cosine and Tangent

Sine (pronounced like 'sign'), Cosine and Tangent are called trig functions. Very simply they are just the different ratios you can form from the sides of the triangle. How do we find their values?

1. Choose an angle in the triangle (e.g. the one labelled θ in the triangle above).
2. Label the sides of the triangle: the longest side we'll call 'c'. The side opposite the angle we'll call 'a'. The other side, we'll call 'b'.
3. In the above right-angled triangle, the ratios of the sides define the sine, cosine and tangent functions (have a look on your calculator – they're abbreviated to 'sin', 'cos' and 'tan').

$$\sin\theta = \frac{a}{c} \quad \cos\theta = \frac{b}{c} \quad \tan\theta = \frac{a}{b}$$

We can also rearrange these into useful combinations.

$$a = c\sin\theta \quad b = c\cos\theta \quad a = b\tan\theta$$

If we have the lengths of the sides of the right-angled triangle but don't know the angle we can use the inverse trig functions labelled 'arcsin' or '$\sin^{-1}$' on your calculator (there are similar functions for inverse cos and inverse tan). If you haven't met these before and want to practice then try the questions below – otherwise skip to **Silent Witness tool 1: SHADOWS**.

Make sure your calculator is in **degrees** mode.

Practice with trig (optional)

1. In the diagram above a = 3 b = 4, what are the values of θ and c?

Answer: We're told a and b so we can form the ratio a/b. This is called tan θ.

Since a is 3 and b is 4, then a/b = 0.75. So we need to find the angle θ which has tan θ = 0.75. To do this we use the tan-1 button on your calculator (ask if you need help). You should get the answer

$$\rightarrow \quad \theta = \tan^{-1}(0.75) = 36.9^o$$

Now to find the long side you can either use Pythagoras' Theorem or another trig function. Try both ways, the answers are given below:

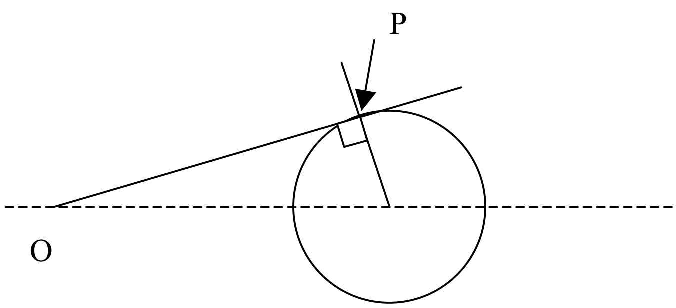
$$c = \frac{a}{\sin\theta} = \frac{3}{\sin 36.9^o} = 5$$

$$c = \frac{b}{\cos\theta} = \frac{3}{\cos 36.9^o} = 5$$

$$or \quad c^2 = a^2 + b^2 = 3^2 + 4^2 = 25 \quad \rightarrow \quad c = 5$$

2. Now let b = 12 c = 13. Show that a = 5 and θ is 22.6°.

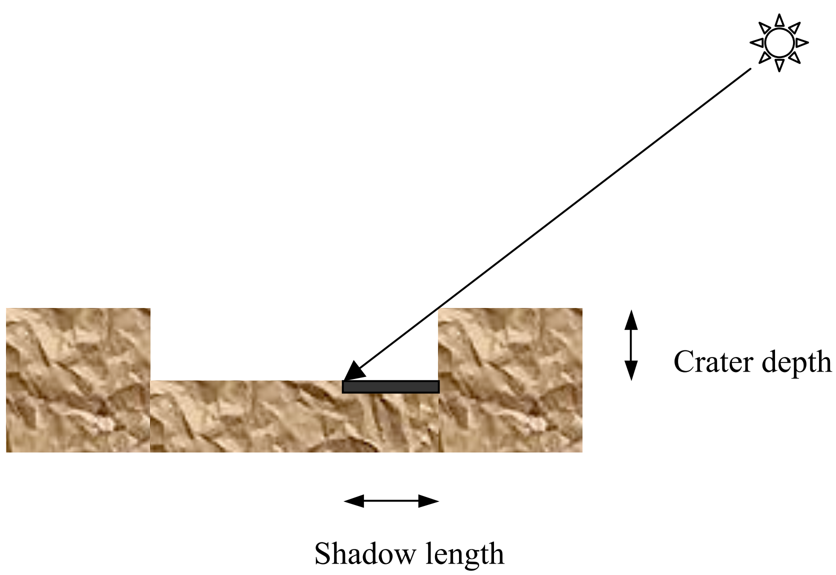
3. In the figure below, the radius of the circle is 7cm and the distance from the centre of the circle to the point O is 25 cm. What length of string is required when pulled taut to reach from O to the tangent point P? Notice that you get a right angle at a tangent point between the tangent and the radius to that point. (hint: use Pythagoras; answer: 24 cm)



Silent Witness tool 1: SHADOWS

Challenge 1: Take a look at the image below. Roughly what height was the Sun in the sky when the photograph was taken? (By height, we mean what angle did the Sun make with the horizon?)

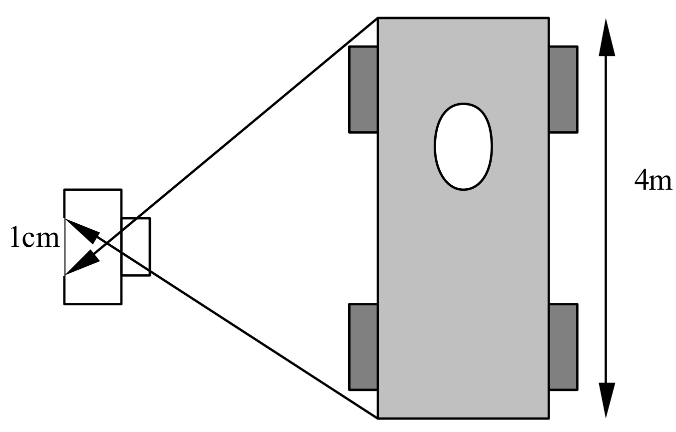
Challenge 2: Take a look at the image below taken from a spacecraft image of Moon. If the diameter of the crater is 10 km and the Sun was ten degrees above the horizon on the Moon, what can you deduce about the depth of the crater?



Silent Witness tool 2: BLURRING

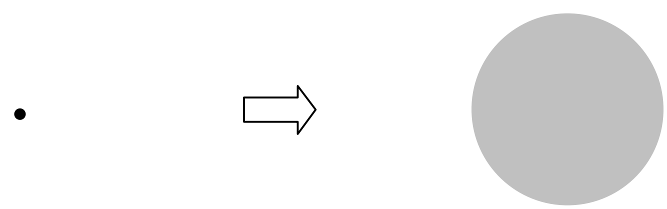
Challenge 3:

A photographer wants to take sharp pictures of a Formula 1 racing car without moving the camera. The car moves at 200 km/hr. The set-up is illustrated below. Two light rays from the ends of the car at one instant in time are drawn. The car is 4m long and gives an image 1cm in length on the film. A picture is blurred if the image moves by more than 0.1mm during the exposure. What is the longest exposure the photographer can use? (**Hint**: First convert 200km/hr to a speed in m/s – you should get 55.56 m/s. Now if the length of the car is 4m and this scales to 1cm on the film, what length on the road corresponds to 0.1 mm on the film i.e. 1/100th the length of the car? Using these bits of information find the time)

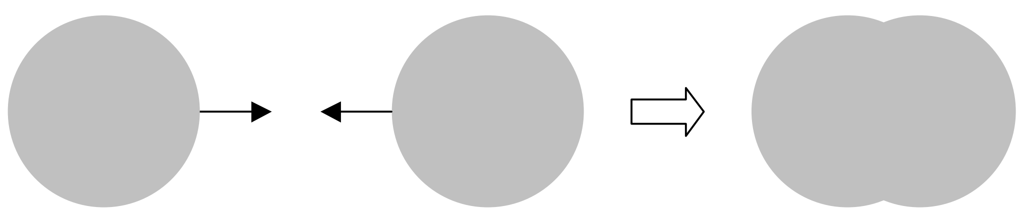


Challenge 4: Sometimes images are blurred because the object is moving e.g. the number plates on a getaway car caught speeding away from a crime site. However no image is absolutely ‘sharp’ even when the object is stationary. This is caused by the lenses and mirrors out of which a camera say is made and the fact that light is a wave – the bigger the lens the smaller the effect – that’s one reason why astronomers build larger and larger telescopes. You’ll also notice when you have an eye test that the smaller letter all appear as blobs without holes and sharp edges. We’ll now find out why.

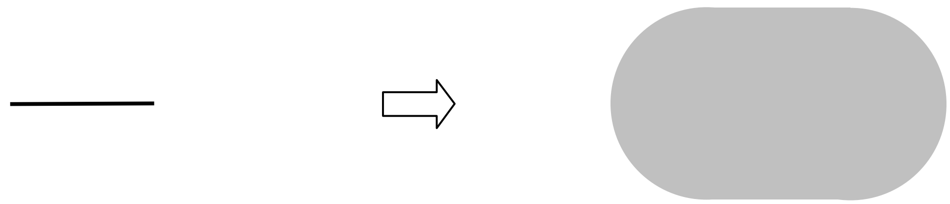
To investigate this, imagine your camera was making an image of the following symbol ϑ (there are three large copies of the symbol below).
We will simplify the real situation and suppose the effect of the camera lenses is to make every black point in the image into a uniform disc i.e.



We will assume for simplicity that if you overlay two grey discs from two points, they don’t get any darker so that



Convince yourself that when the camera makes an image of the line below, the picture on the film will look like the stadium shaped grey blob on the right hand side.



Now we will try to find the blobs (images on the camera film) that correspond to our more complicated shape – the symbol Ə. To do this, choose one of the washers and try drawing round the washer, positioning the centre of the washer on the edge of the symbol. The centre is in the hole, so you will have to make the best guess you can. Move the washer around the inner and outer edge until you have enough arcs to see the rough shape of the edge of the blob. Repeat on the copies of the picture with different sized washers. What conclusions can you draw? The bigger the washer, the smaller the camera lens. Which camera lens allows you to see most detail? Would you still recognise the symbol on the picture or could you confuse it for another symbol?

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Silent Witness tool 3: PERSPECTIVE

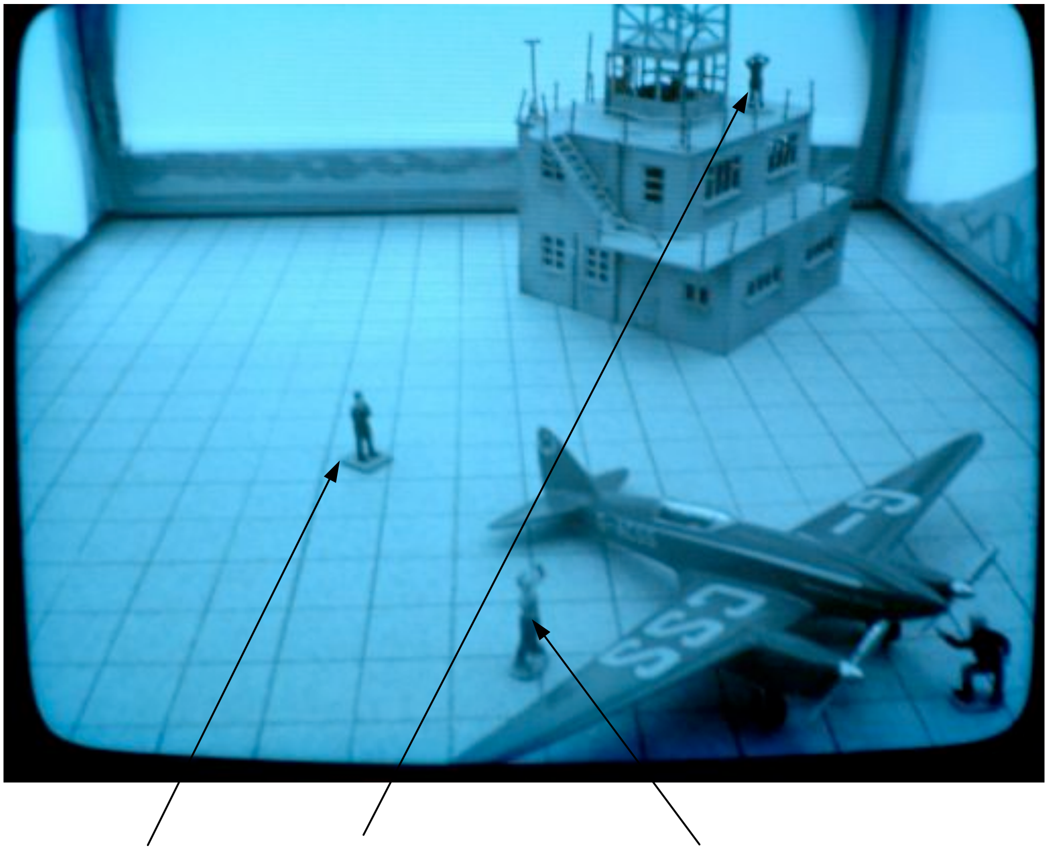
Now we move into the world of movie making.

Challenge 5: A sequel is planned to ***A Bug's Life***. An ant comes across an upside-down can (see picture below; you'll need to use the actual can in the Masterclass).

How close can the ant get to the tin before it cannot see all the <u>complete</u> letters of **TESCO**?

Challenge 6: A film is being made about the Comet Racer – if you've ever been inside the Galleria or looked outside the Comet Hotel in Hatfield, you will have seen models of this aircraft. The director has a model of one of the sets made up and uses a small TV camera to picture the scene. Take a look on the monitor in the Masterclass to see the scene – you can use the laminated blue screen shots in this exercise or the picture below.

To help plan the layout, the model 'floor' has been marked out in squares of side 1.2 metres. Some of the model characters are illustrated



Using the TV image/picture, try and answer the following questions – be as exact as you can and give your answers in the units that will be required for the real set i.e. metres or square metres:

1. What is the distance between the photographer and the mechanic?

2. What is the distance between the photographer and the wing commander?

3. What is the ground floor area of the control tower building?

4. **Tricky one**: How high is the camera off the ground in this shot? (Hint – use the squares and look at how they become 'thinner' at large distances. How is that 'thinness' related to the angle at which you view the square? With two different squares along the central line-of-sight and two angles for each, you should be able to attempt a scale drawing.

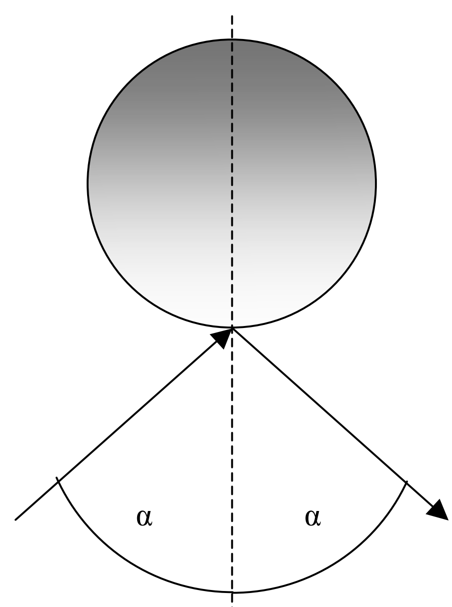
Silent Witness Extra – for the brave!

Silent Witness tool 4: REFLECTIONS

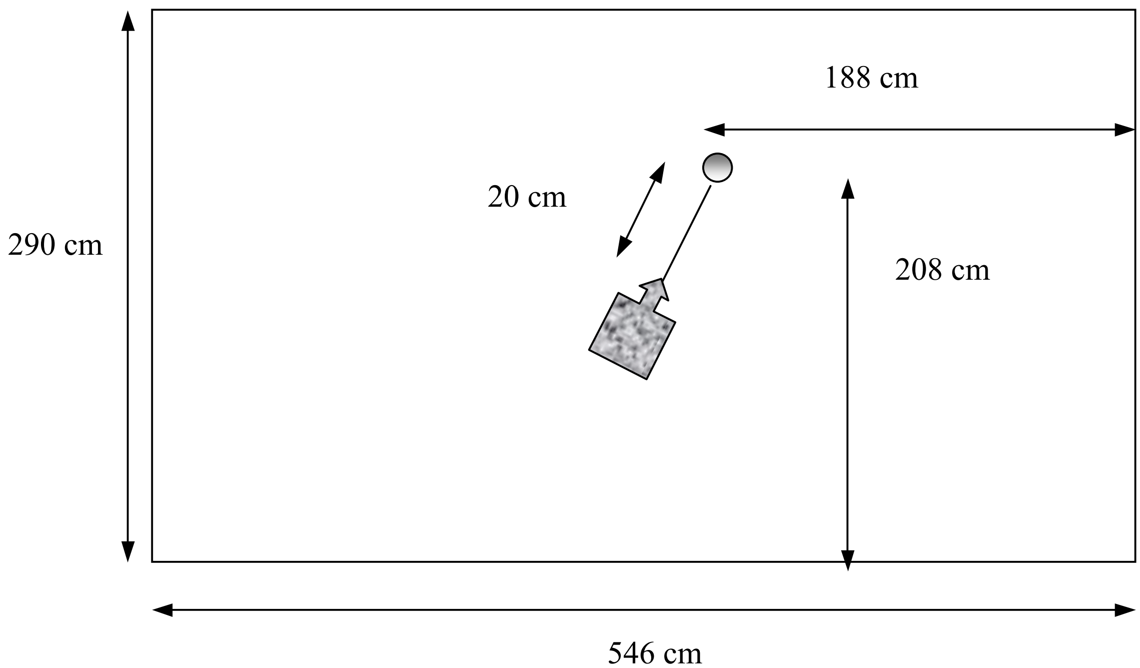
One of the interesting tricks being used in the movie world involves the combination of live-action and animated characters. To make the scenes as realistic as possible the animated characters have to be rendered so that they appear illuminated by the same lighting as the real characters. A novel trick being used is to use the reflections in the eyeballs of the real characters to deduce the position of the light sources.

In this tough exercise, we will do something similar but rather than using an eyeball, we will use a mirror sphere (like a decoration from a Christmas tree). The game is to measure the positions of the lights in the picture of the sphere and then project them into the room. Concentrate on the two lamps on either side of the photographer's head.

You will need one new fact – the law that tells us how light rays are reflected when they hit the sphere. The angle of reflection is equal to the angle of incidence – see the diagram below. The dotted line passes through the centre of the sphere.



The room and the position of the sphere  and camera  are illustrated below.
The line between the camera and sphere is inclined at 20 degrees to the shorter wall



The sphere is 6cm in diameter. **You are told that the lamps lie as close to the wall as they can** and the radius of the lampshade is 25cm. You have to find the position of the two lamps in the room.

(You can assume the two lamps, the sphere and the camera are all at the same height)

Hint: measure the distance of the image of a lamp from the centre of the sphere. Use the distance from the sphere to work out the angle of reflection. On a scale drawing, plot the light rays from the lamp and hence deduce the position of the two lamps.