Two-Component Traffic Modelled by Cellular Automata: Imposing Passing Restrictions on Slow Vehicles Increases the Flow

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Abstract. We use a computer-based cellular automaton to study two-component flow, mimicking (fast) passenger and (slow) cargo vehicles, on a circular unidirectional two-lane highway without on-ramps and exits. The global flow rates for different overall densities and mixing ratios between fast and slow cars are determined. We study two main scenarios: two-component traffic without passing restriction (uncontrolled flow) and traffic in which slow vehicles are prohibited to pass (controlled flow). We find that controlling the flow should considerably increase multi-lane highway capacity.

Keywords: Traffic flow, cellular automata, environmental regulation.

1 Introduction

Modelling road traffic behaviour using cellular automata has become a well-established method to model [1–3], analyze [1, 3–5], understand [1–6] and even forecast [3, 7] the behaviour of real road traffic [7, 8].

A well-established and popular cellular automaton model is due to Nagel and Schreckenberg [1, 3]. It is known to be 'minimal' [6] in the sense of containing just the necessary rules to simulate realistic phenomena such as the spontaneous formation of jams on busy roads [6], throttling of traffic flow on busy roads [8], and the spontaneous emergence of density waves [7, 9].

Although exact analytical results for this [1] and related systems [7] are typically not available [3, 4, 7, 10], the automata's evolution rules are simple, straightforward to understand, computationally efficient and sufficient to emulate much of the behaviour of observed traffic flow [8]. Cellular automaton traffic simulations of the Nagel-Schreckenberger-type have thus proven useful and popular [1–7, 11].

Here, we present results of such a simulation for two-component traffic on a two-lane highway which is closed to a loop without on-ramps and exits [11]: our computational model is defined by a two dimensional array (number of lanes) of L sites (position on road of length L). This setup was chosen for its simplicity.

Each site may either be occupied by one vehicle, or is empty. We assume the two-component traffic to consist of faster cars and slower (transport) vehicles with different

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attainable maximal speeds $v_{max,F} > v_{max,S}$. This is the only characteristic that distinguishes the two types of vehicles. For simplicity, we do not include other realistic effects, such as effects due to different vehicle lengths, widths, and their different acceleration rates, for instance.

After a presentation of our automaton's evolution rules in section 2 we reproduce the fundamental diagrams known from previous works [1–7, 11] in section 3. In section 4 we study traffic flow under a passing restriction for slow vehicles which displays our main result that prohibiting slow vehicles from passing leads to increased flow and hence increased capacity of multi-lane highways. In section 5 we present an outlook on future work and our conclusions.

2 The Evolution Rules

For simplicity and for the sake of easy comparison with published work we employ only the four necessary evolution rules laid down by Nagel and Schreckenberger, and simply cite from their work [1]:

"Each vehicle has an integer velocity with values between zero and v_{max}. For an arbitrary configuration, one update of the system consists of the following four consecutive steps, which are performed in parallel for all vehicles:

1) **Acceleration:** if the velocity v of a vehicle is lower than v_{max} and if the distance to the next car ahead is larger than $v+1$, the speed is advanced by one $[v \to v+1]$.
2) **Slowing down (due to other cars):** if a vehicle at a site i sees the next vehicle at site $i+j$ (with $j \leq v$), it reduces its speed to $j-1$ $[v \to j-1]$.
3) **Randomization:** with probability p, the velocity of each vehicle (if greater than zero) is decreased by one $[v \to v-1]$.
4) **Car motion:** each vehicle is advanced v sites."

In modelling a highway, we treat unidirectional *two-lane* traffic, rule '2)' therefore has to be modified by a suitable rule that allows cars to avoid a slower car ahead of them by changing lanes. Accordingly [2], we devise the alternative *passing rule*:

$\bar{2}$) **Lane change or slowing down (due to other cars):** if a vehicle at a site i sees the next vehicle at site $i+j$ (with $j \leq v$), it changes lanes or, if blocked by a third car k in the other lane at a distance smaller than that car's speed $v_k > i-k$, reduces its speed to $j-1$ $[v \to j-1]$.

Note, that this passing rule describes *considerate drivers*: the lane changing step is only executed if cars do not force approaching traffic to slow down. We chose this implementation of the passing rule in order to avoid unrealistically confrontational lane changing behaviour. An inconsiderate lane changing behaviour would, moreover, unduly exaggerate the possible flow improvements due to imposing passing restrictions on slow traffic that we are studying here.

The above rules imply equal probabilities for lane changes, we therefore arrive at a symmetric distribution of traffic across both lanes if no flow control is imposed (for lane-population inverting mechanisms see [2, 3]).

3 Two-Component Traffic without Passing Restriction

In order to quantify our results we consider the following global quantities:

$$\text{average speed} \quad V = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=S,F} v_{i,j,near} + v_{i,j,far} \tag{1}$$

$$\text{overall density} \quad \rho = \rho_S + \rho_F = \frac{N_S + N_F}{2L} \leq 1 \tag{2}$$

$$\text{and total flow} \quad J = \rho V \, , \tag{3}$$

where the indices S and F refer to slow and fast vehicles, *near* and *far* refer to the two lanes which also gives rise to the normalization factor $\frac{1}{2L}$ for the flow. The counting index i over the total number of cars $N = N_S + N_F$ on the modelled road of length L provides us with a *global average along the entire road*. This circumvents some subtle problems associated with the biases of various local flux measures [7].

Note that our definition of the flow J does not discriminate between fast and slow cars. One could argue that slow cars should have a large weighting because they tend to transport more material. One could equally argue they should count less since they effectively only move material as opposed to people, we therefore decided to give them equal weight to the fast cars.

3.1 Slow Vehicles Only

It is well known [1, 6, 7] that traffic flow shows two main phases: the homogeneous flow phase for low traffic density ρ in which the overall flow J_H is proportional to traffic density and effective maximum speed. Since the effective maximum speed is given by $\hat{v} = v_{max} - p$ with p the random deceleration probability introduced in evolution rule '3)', we find [6]

$$J_H = \rho(v_{max} - p) \, . \tag{4}$$

The other phase corresponds to the jammed state with the flow [6]

$$J_J = (1-\rho)(1-p) \, . \tag{5}$$

This expression can be understood as the product of the remaining free road $(1-\rho)$ [with the tacit assumption that for a very congested road most traffic is trapped in a jam, i.e. $\rho_{flowing} \approx (1-\rho)$] and the probability for a vehicle to emerge from the front of a jam, i.e. the drive-off probability $1-p$. The jam thus acts as a continuous reservoir determining the vehicle flux, expression (5) is consequently independent of the average maximal velocity $\hat{v} = v_{max} - p$.

Fig. 1. confirms the above flow expression for J_H and J_J; see dotted lines in the plot. It, moreover, shows a comparison between single and two lane highways. In this context, let us emphasize that the normalization constant of the density for the single lane case is, of course, given by $\frac{1}{L}$ rather than $\frac{1}{2L}$ as in the two-lane expression (2) and the entire rest of this paper. We kept all other conditions identical and find that in the two-lane case the global flow rate is higher than for one-lane highways because vehicles can somewhat avoid an impasse by lane changes, see Fig. 1.

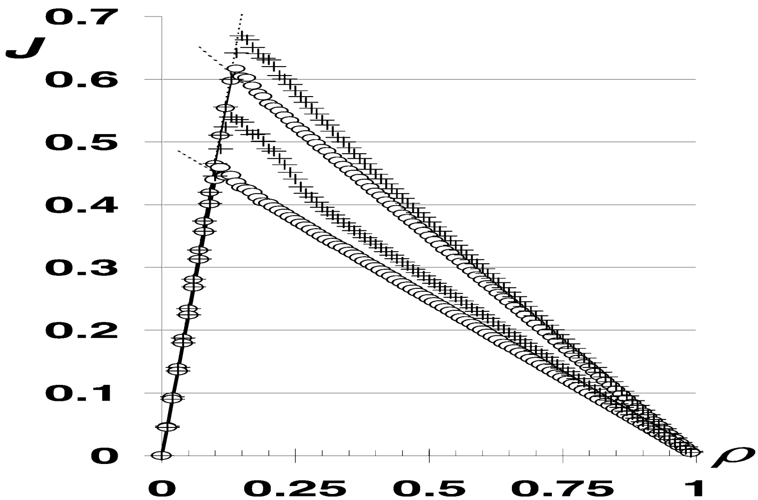

Fig. 1. Fundamental diagram plots of the relative flow J as a function of the global road occupation density ρ for one component traffic (slow vehicles only) with $v_{max} = 5$ and random deceleration probabilities $p = 0.3$ (higher curves) and $p = 0.5$ (lower curves). Compared are the cases single lane road 'o' and two lane road '+'. The dotted lines represent the theoretical curves derived for free flow J_H and jammed flow J_J on *one-lane highways*, see Eqs. (4) and (5).

3.2 Fast Vehicles as Well

The effect of adding faster cars (with $v_{max} = 10$) leads to increased flow in the free-flow regime but once the traffic starts to jam this difference diminishes. This effect is displayed in FIG. 2. It shows the fundamental diagram for two-component traffic with two different mixing ratios between slow and fast vehicles. Only one random decceleration probability $(p = 0.3)$ is employed in this plot.

One finds that the homogenous flow region now shows a new transition. At very low densities the flow is homogenous for both slow and fast vehicles

$$J_{HH} = \rho_F \hat{v}_F + \rho_S \hat{v}_S \; ; \tag{6}$$

here, in accordance with equation (4), $\hat{v} = v_{max} - p$ is the average maximal velocity. At slightly higher densities the much lower flow regime $J_{JH} = \rho \hat{v}_S$, dominated by the free flow of slow vehicles only, takes over. This is due to the fact that the faster vehicles jam whenever a bottleneck forms because one slow vehicle passes another [2]. It is this very observation on which our work focusses and from which we derived the idea that slow vehicles have to be controlled (have to be prohibited from passing) in order to increase multi-lane highway capacities. We, therefore, now turn our attention to the case of such 'controlled flow'.

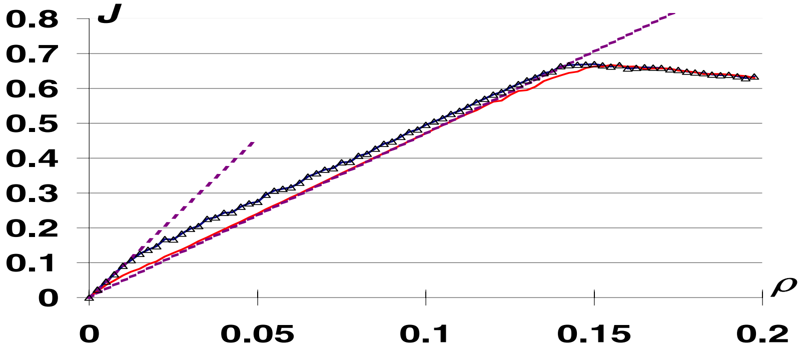

Fig. 2. Fundamental plots of the relative flow J as a function of the road occupation density ρ for two-component traffic on a two-lane road with $v_{max,S} = 5$ and $v_{max,F} = 10$, and random deceleration probability $p = 0.3$. Displayed are the cases $\rho_F/\rho_S = 9/1 \hat{=} 90\% : 10\%$ and $\rho_F/\rho_S = 3/2 \hat{=} 60\% : 40\%$, namely top line with data points labelled '△' and red lower line. The two dotted lines describe the flow rates J_{HH} and J_{JH}, see Eq. (6) and text thereafter.

4 Controlled Flow: Slow Vehicles Must Not Pass

We want to compare the cases studied in the previous section with a scenario where slow (cargo) vehicles are not allowed to pass, i.e., are restricted to the 'near' lane. The corresponding rules of our model are therefore modified as follows:

Firstly, the initial distribution of slow vehicles is confined to the near lane. Note, that we always define densities ρ with respect to the entire road surface ($2L$ cells) just as in Eq. (2) above. The respective density distributions are $\rho_S = \rho_{S,near} = 0, \ldots, \frac{1}{2}$ and $\rho_{S,far} = 0$. Consequently, the occupancy $\sigma_{S,near} = N_{S,near}/L$ of slow cars in the near lane is their overall density doubled: $\sigma_{S,near} = 2\rho_S = 0, \ldots, 1$; this obviously leads to the upper limit $\rho_S = \frac{1}{2}$. For the fast cars we chose an unbiased lane occupation ratio proportional to the remaining space, that is $\rho_{F,near} = \rho_F \cdot (1 - \sigma_{S,near})/(2 - \sigma_{S,near})$ and $\rho_{F,far} = \rho_F \cdot 1/(2 - \sigma_{S,near})$, since we define densities with respect to the entire road surface this implies $\rho_{F,near} + \rho_{F,far} = \rho_F$.

Secondly, and more importantly, slow vehicles are not allowed to pass, so the 'no-passing rule' is implemented by substituting evolution rule $\overline{2}$) by rule 2) for slow vehicles only (the fast ones are still allowed to change lanes) thus *confining all slow vehicles to the near lane*.

The imposition of the no-passing rule for slow vehicles shows a considerable increase of the global flow rate (see Fig. 3 two top curves) as compared to the case of slow vehicles being allowed to pass other vehicles (see figure 3 two bottom curves, or the same two curves in figure 2). For further quantification Fig. 4 displays the relative difference of the increase in flow rate defined as

$$\Delta j \doteq \frac{J_{\square}}{J} - 1 \,. \tag{7}$$

One can see from Fig. 4 that, with our choice of parameters, the introduction of the no-passing rule for slow vehicles at a mixing ratio $N_F/N_S \hat{=} 90\% : 10\%$ increases the relative flow by up to 55%.

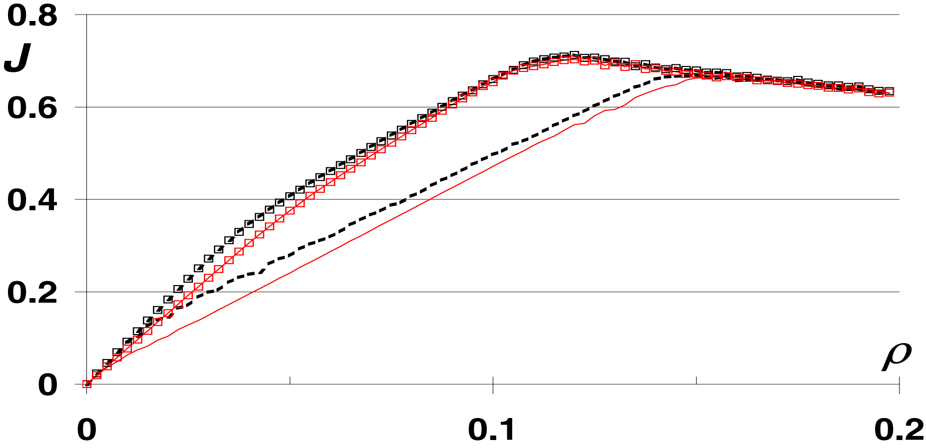

Fig. 3. Same as figure 2 (lower two lines) ($v_{max,S} = 5$ and $v_{max,F} = 10$) plus additionally scenario '□' with same parameters but slow vehicles now obey no-passing rule (two top lines) thus considerably increasing the flow. Vehicle mixing ratios $\rho_F/\rho_S = 9/1 \mathrel{\widehat{=}} 90\% : 10\%$ for black broken lines and $\rho_F/\rho_S = 3/2 \mathrel{\widehat{=}} 60\% : 40\%$ for thin red solid lines.

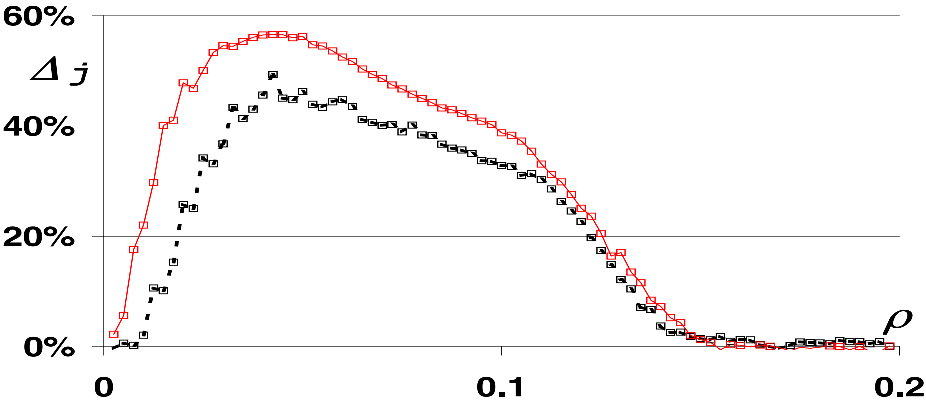

Fig. 4. Relative difference of the flow rates displayed in Fig. 3: $\Delta j = (J_\square/J) - 1$. Vehicle mixing ratios $\rho_F/\rho_S \mathrel{\widehat{=}} 90\% : 10\%$ for lower broken line and $\rho_F/\rho_S \mathrel{\widehat{=}} 60\% : 40\%$ for top thin red solid lines.

Even in the case of a much smaller differential of $v_{max,F}/v_{max,S} = 10/8$ (rather than 10/5 considered before) for the maximum-speeds we still witness a significant enhancement of the global flow rate by more than 10%, see Fig. 5.

5 Outlook and Conclusions

We expect that modifications of the model presented here should typically *amplify* the beneficial effects of confining slow (cargo) traffic. Such modifications to our model

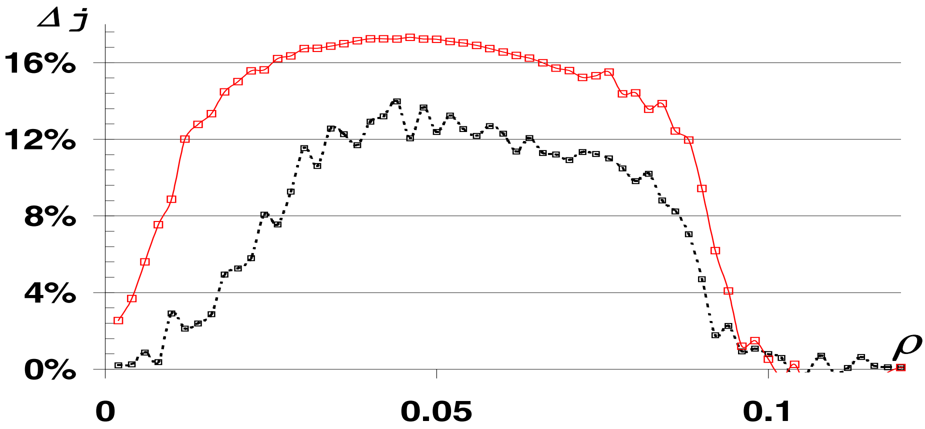

Fig. 5. Relative difference of the flow rates $\Delta j = (J_{\square}/J) - 1$, just as displayed in Fig. 4, but with a smaller maximum-speed differential, namely $v_{max,S} = 8$ and $v_{max,F} = 10$. Vehicle mixing ratios $\rho_F/\rho_S \hat{=} 90\% : 10\%$ for lower dotted line and $\rho_F/\rho_S \hat{=} 60\% : 40\%$ for top thin red solid lines. The residual noise due to finite size effects in our simulation is clearly visible in these plots.

could include slower acceleration for the slow and heavy (cargo) vehicles. These are typically also longer than light and fast (passenger) vehicles thus reducing the available road space. Since it takes a longer time to pass a long vehicle, particularly if another slow and slowly accelerating long vehicle is passing, the inclusion of these two straightforward modifications should further emphasize the beneficial effects of barring slow vehicles from passing.

One could also change the composition of the traffic in order to simulate more realistic multi-component traffic and modify the evolution rules to include technical and psychological effects [6, 7]. In either case we believe the considerable increase in road capacity of multi-lane highways due to the restriction of slow (cargo) vehicles from passing should persist.

We conclude that one should seriously consider to perform field trials to establish whether slow (cargo) vehicles should always be restricted to the 'near' lane in order to increase road capacity of multi-lane highways at no extra cost.

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