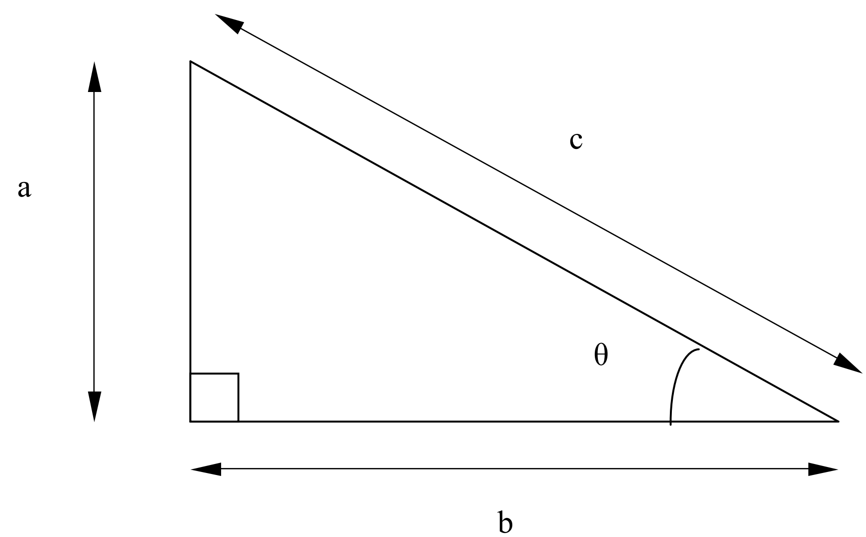
Silent Witness Answers – Using Mathematics to Decode Images



Make sure your calculator is in **degrees** mode.

Practice with trig (optional)

1. In the diagram above a = 3 b = 4, what are the values of θ and c?

Answer: We're told a and b so we can form the ratio a/b. This is called tan θ.

Since a is 3 and b is 4, then a/b = 0.75. So we need to find the angle θ which has tan θ = 0.75. To do this we use the tan-1 button on your calculator (ask if you need help). You should get the answer

$$\rightarrow \quad \theta = \tan^{-1}(0.75) = 36.9^o$$

Now to find the long side you can either use Pythagoras' Theorem or another trig function. Try both ways, the answers are given below:

$$c = \frac{a}{\sin\theta} = \frac{3}{\sin 36.9^o} = 5$$

$$c = \frac{b}{\cos\theta} = \frac{3}{\cos 36.9^o} = 5$$

$$or \quad c^2 = a^2 + b^2 = 3^2 + 4^2 = 25 \quad \rightarrow \quad c = 5$$

2. Now let b = 12 c = 13. Show that a = 5 and θ is 22.6°.

Answer: We're told *b* and *c* so we can use Pythagoras to find *a*.

$$c^2 = a^2 + b^2 \quad 13^2 = a^2 + 12^2 \quad \rightarrow \quad a = 5$$

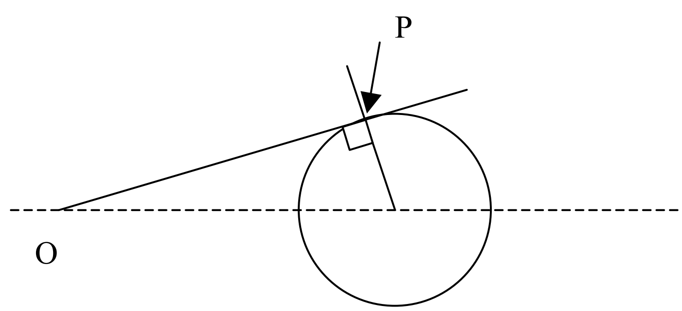
then use any one of the following

$$\begin{aligned} \theta &= \tan^{-1}(5/12) = 22.6^o \\ \theta &= \sin^{-1}(5/13) = 22.6^o \\ \theta &= \cos^{-1}(12/13) = 22.6^o \end{aligned}$$

3. In the figure below, the radius of the circle is 7cm and the distance from the centre of the circle to the point O is 25 cm. What length of string is required when pulled taut to reach from O to the tangent point P? Notice that you get a right angle at a tangent point between the tangent and the radius to that point. (hint: use Pythagoras; answer: 24 cm)

Answer: We're told *a=7* and *c=25,* so we can use Pythagoras to find *b*.

$$c^2 = a^2 + b^2 \quad 25^2 = 7^2 + b^2 \quad \rightarrow \quad b = 24$$



Silent Witness tool 1: SHADOWS

Challenge 1 Answer: You could draw a line joining the end of the shadow to the top of the post and then use a protractor or form a triangle. You should get an answer of about 57 degrees.

Challenge 2 Answer: Measure the crater diameter on the picture. This is 15mm. The shadow length is 5mm so the width of the crater shadow on the Moon is 10/3 = 3.333 km. Then use:

$$\tan(10^o) = \frac{crater\ depth}{shadow\ length} \quad \rightarrow \quad depth = 3.333\tan(10^o) = 0.59km$$

Silent Witness tool 2: BLURRING

Challenge 3 Answer:

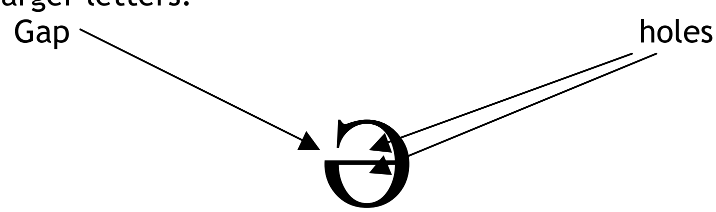
First convert 200km/hr to a speed in m/s

$$speed = 200 \times 1000 \times \frac{1}{60 \times 60} = 55.56 m/s$$

Now the length of the car is 4m and this scales to 1cm on the film. The length on the track that corresponds to 0.1 mm on the film i.e. 1/100th the length of the car is 4/100= 0.04m (i.e. 4cm). The car will move 0.04m in a time

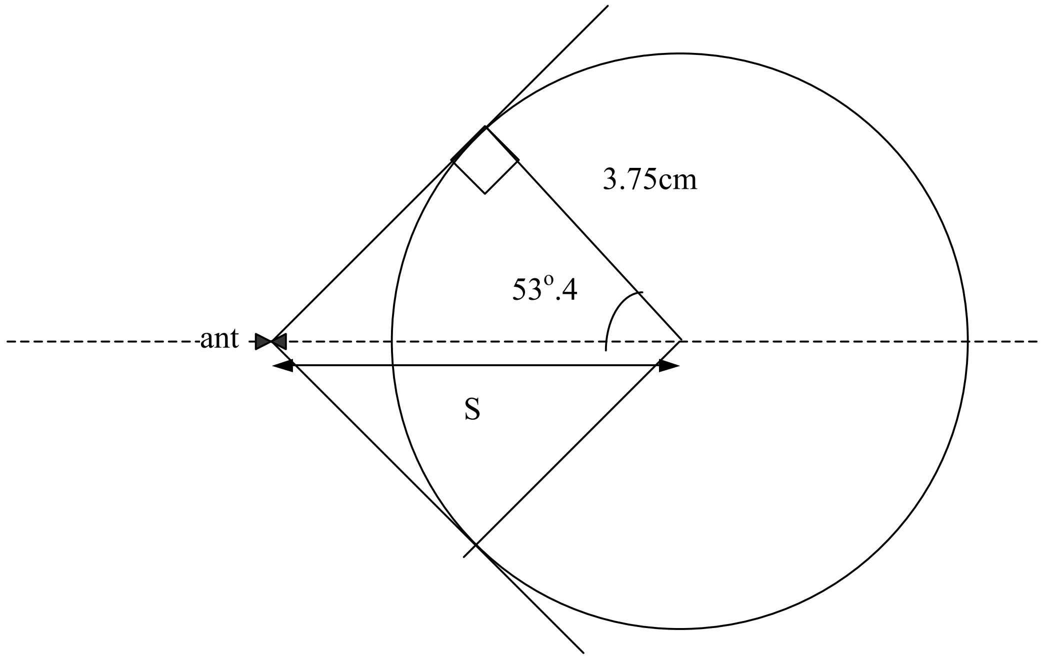
$$time = \frac{0.04}{55.56} = 7 \times 10^{-4} s \approx \frac{1}{1000} s$$

Challenge 4 Answer: With the largest washer, you will find that the holes in the symbol are filled in and the whole symbol becomes a roughly circular blob. With the smaller washers, the inner spaces are seen but the gap in the symbol is lost. You can think of your eye as a washer so that when you try and view small letters at the opticians, they become blobs whilst you can make out the details in the larger letters.



Silent Witness tool 3: PERSPECTIVE

Challenge 5 Answer: Measure the length of the word **TESCO** by wrapping a piece of paper round the can. You should find you get 7cm. The diameter of the tin is 7.5cm, so the circumference is $\pi D = 23.6cm$. The angle subtended by the word **TESCO** at the centre of the tin is therefore, $\frac{7}{23.6} \times 360 = 106^o.8$. We can then draw the following figure. Note how the ant's limit of vision around the tin is given by the two tangents.



From the figure, $\cos(53^o.4) = \dfrac{3.75}{S} \quad \rightarrow \quad S = 6.29cm$. This means that the ant is (6.29 – 3.75) cm away from the can i.e. 2.6cm

Challenge 6 Answer: (note that these are approximate and your answers may be slightly different; 1 grid square unit = 1.2m)

1. What is the distance between the photographer and the mechanic?

   Use Pythagoras: in units of grid squares
   $d^2 = 2.4^2 + 4.6^2 \quad \rightarrow \quad d = 5.2\, grid\ units = 6.2m$

2. What is the distance between the photographer and the wing commander?

   First find the height of the control tower using the grid square closest to the leftmost edge on the picture. You should get a height of ~ 4 grid units or 4.8m. Draw lines down from the Wing Commander and guess where it intersects the base. Then use Pythagoras to find the distance between this point and the cameraman: $d^2 = 7.4^2 + 7.9^2 \quad \rightarrow \quad d = 10.8\, grid\ units = 13m$. So it's about 13m on the base and 4.8m up, so using Pythagoras again:
   $d^2 = 13^2 + 4.8^2 \quad \rightarrow \quad d = 13.8\, grid\ units = 16.6\, m$

3. What is the ground floor area of the control tower building?

Use Pythagoras on both visible sides: in units of grid squares

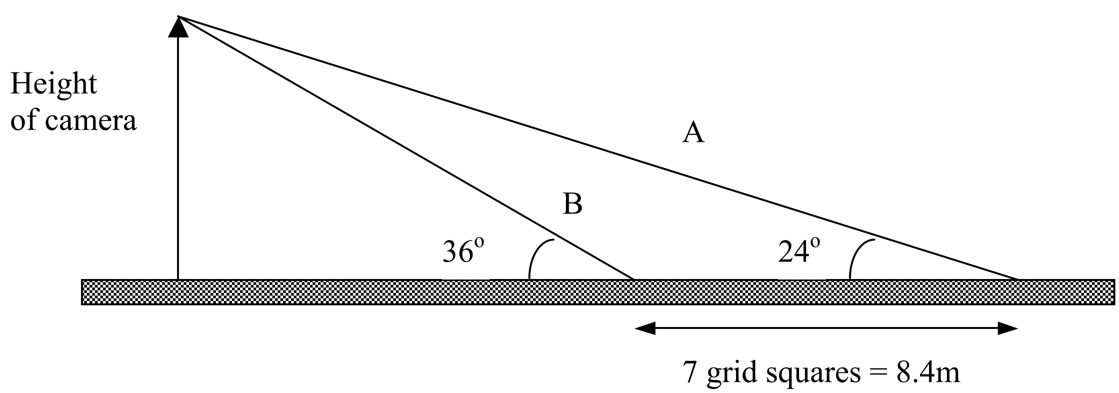
$r^2 = 3.1^2 + 3.8^2 \quad \rightarrow \quad d = 4.9\, grid\ units = 5.9m$

$l^2 = 3.2^2 + 3.7^2 \quad \rightarrow \quad d = 4.9\, grid\ units = 5.9m$

So the building is square and has ground floor area 5.9 x 5.9 =34.8m^2.

4. **Tricky one**: How high is the camera off the ground in this shot? (Hint – use the squares and look at how they become 'thinner' at large distances. How is that 'thinness' related to the angle at which you view the square? With two different squares along the central line-of-sight and two angles for each, you should be able to attempt a scale drawing.

Mark two grid squares along the central line through the TV image. Let's suppose we choose two that are 7 grid squares apart. For each grid square, work out the ratio of the 'vertical' size of the grid square to the 'horizontal' size of the grid square. Take the inverse sine (sin^{-1} on your calculator) of this ratio at each of your chosen grid squares to get two angles; two example angles are marked on the figure below. Then draw a scale diagram and find where the two lines A and B intersect. This gives you an idea where the camera is. (There are other ways too). The answer is about 9.4m.



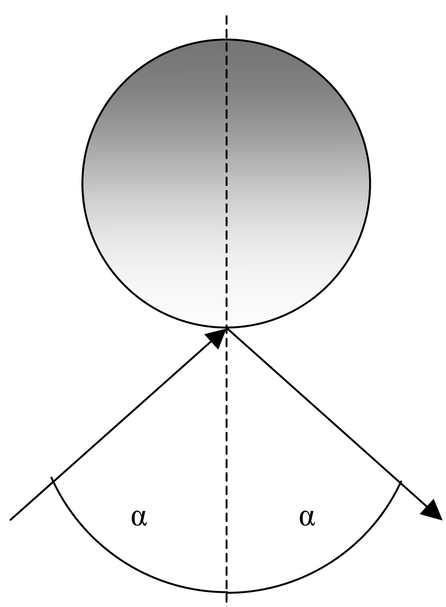
Silent Witness Extra – for the brave!

Silent Witness tool 4: REFLECTIONS

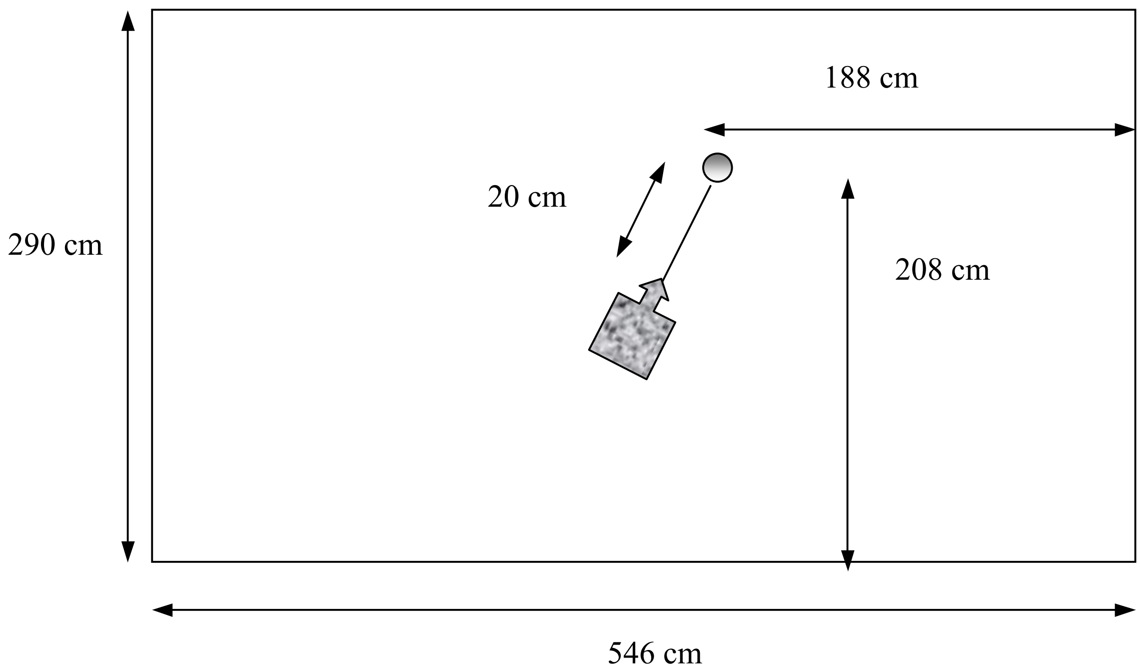
One of the interesting tricks being used in the movie world involves the combination of live-action and animated characters. To make the scenes as realistic as possible the animated characters have to be rendered so that they appear illuminated by the same lighting as the real characters. A novel trick being used is to use the reflections in the eyeballs of the real characters to deduce the position of the light sources.

In this tough exercise, we will do something similar but rather than using an eyeball, we will use a mirror sphere (like a decoration from a Christmas tree). The game is to measure the positions of the lights in the picture of the sphere and then project them into the room. Concentrate on the two lamps on either side of the photographer's head.

You will need one new fact – the law that tells us how light rays are reflected when they hit the sphere. The angle of reflection is equal to the angle of incidence – see the diagram below. The dotted line passes through the centre of the sphere.



The room and the position of the sphere  and camera  are illustrated below.
The line between the camera and sphere is inclined at 20 degrees to the shorter wall



The sphere is 6cm in diameter. **You are told that the lamps lie as close to the wall as they can** and the radius of the lampshade is 25cm. You have to find the position of the two lamps in the room.

(You can assume the two lamps, the sphere and the camera are all at the same height)

Hint: measure the distance of the image of a lamp from the centre of the sphere. Use the distance from the sphere to work out the angle of reflection. On a scale drawing, plot the light rays from the lamp and hence deduce the position of the two lamps.