Lunar Eclipse 3rd and 4th March 2007

Digital photographs from Hertford

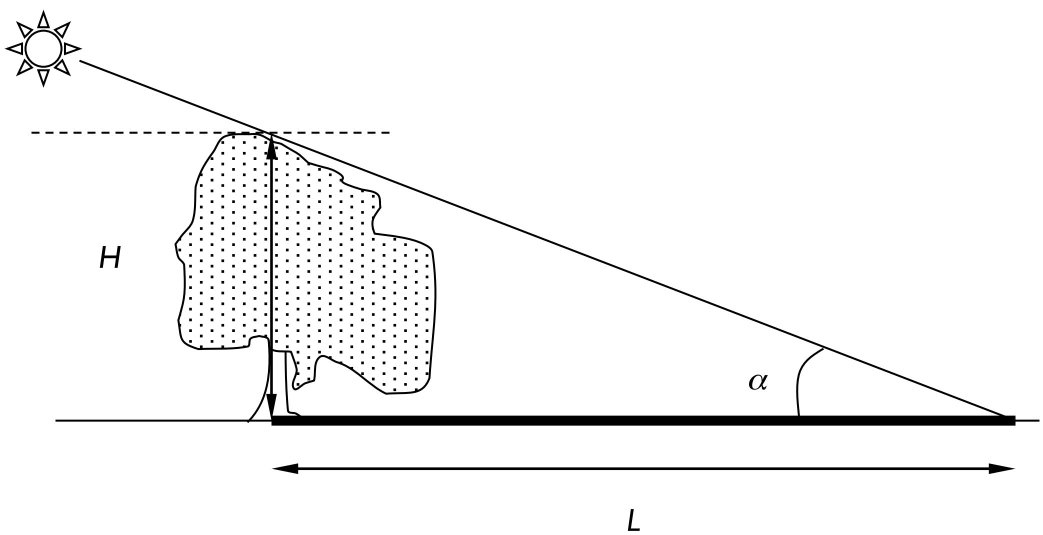
Anticlockwise from top left

20.57 p.m   1/160s   ISO 200
21.36 p.m   1/160s   ISO 200
23.00 p.m   1/5s   ISO 400
23.40 p.m   1/2s   ISO 400
00.24 a.m   1/60s   ISO 400

Shadows

In this session, we will look at the mathematics behind eclipses, in particular a lunar eclipse. Eclipses occur when a portion of the Earth is in the Moon's shadow (a solar eclipse) or the Moon is in the Earth's shadow (a lunar eclipse). Eclipses can be partial or total depending on whether, from the viewpoint of an observer, only a part of the Sun is obscured by the body casting the shadow or all of it.

Shadows are generally useful things. For instance, you can work out the height of a tree by measuring the angle the Sun makes with the horizon and the length of the shadow.



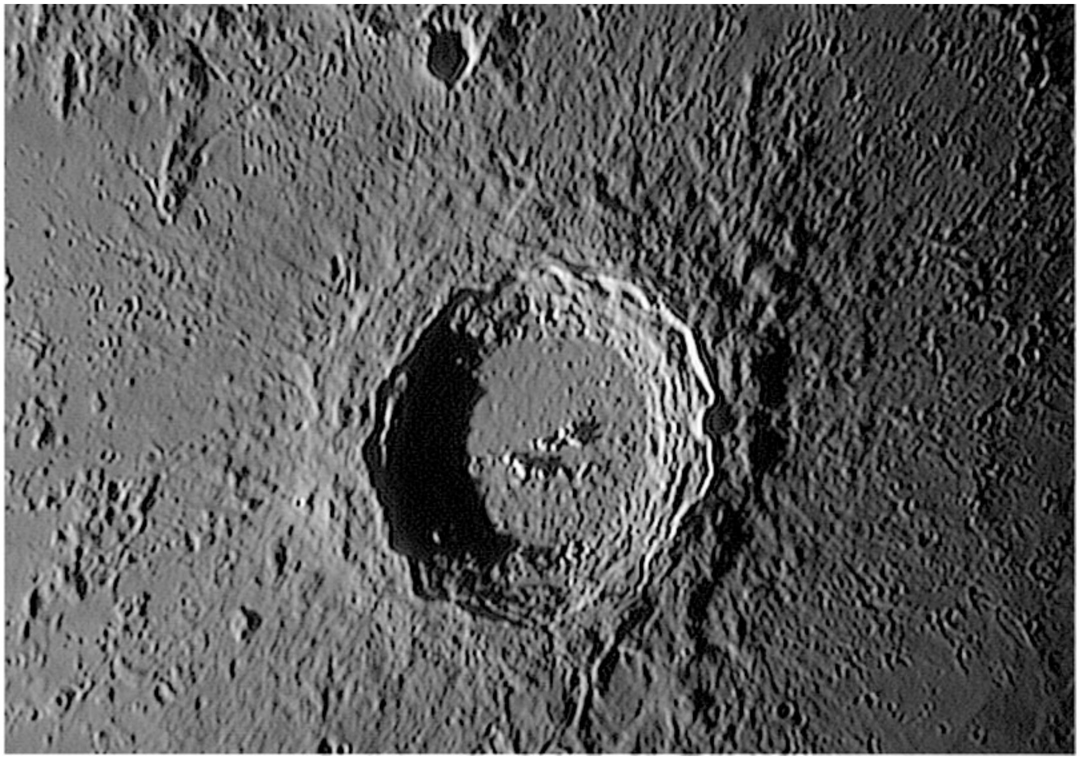
To find the height we make use of a **trig**(onometric) function called the tangent or just **tan**. Tan is a property of an angle. The definition of tan is the ratio of two sides in a right-angled triangle - the side opposite the angle whose 'tan' we're trying to find and the shorter side that makes up the angle. So in our picture:

$$\tan\alpha = \frac{length\ of\ side\ opposite\ the\ angle}{length\ of\ side\ adjacent\ the\ angle} = \frac{H}{L}.$$

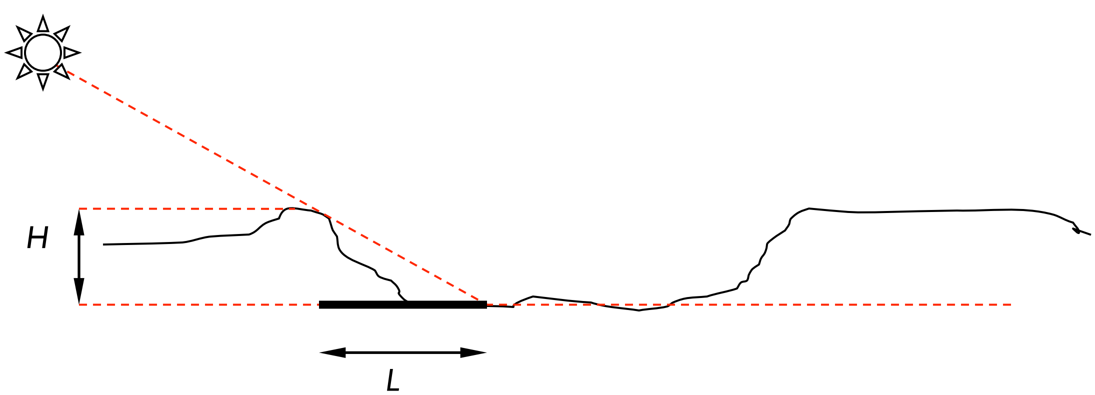
Example: When the Sun was 30° above the horizon, an oak tree cast a shadow that was forty metres long. How tall is the tree?

First press the **tan** button on your calculator and then **30**. You should get 0.57735. So we have $\tan\alpha = 0.57735 = \frac{H}{L}$. Rearranging, we can write $H = L\tan\alpha$. Putting in the length of the shadow *L*, we have $H = 0.57735L = 23.094\,\mathrm{m}$.

Problem 1: How long in theory will the shadow be from a tree 25 m tall just after sunrise when the Sun is low in the sky, 10 degrees above the horizon?

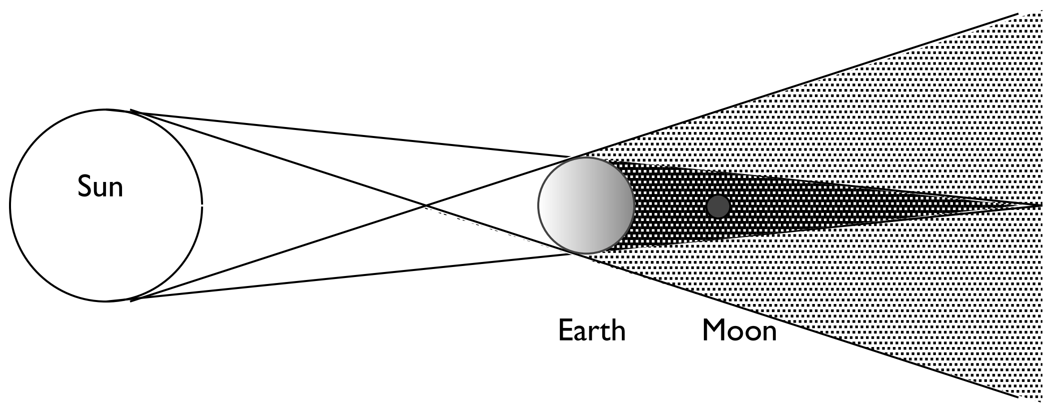


Now that's all very well but it's much more impressive when you can measure something on the Moon. The picture above shows the crater Copernicus (imaged by the US Naval observatory). You can see that the crater walls cast a shadow on the crater floor. We can use this to work out the depth of the crater.



Problem 2: We can measure the diameter of Copernicus from Earth once we know the distance to the Moon. It turns out to be 93 km. The Sun's altitude was 8 degrees. Use the image to work out the length of the shadow on the Moon's surface and hence deduce the depth of Copernicus. (What assumptions have you made?)

The Geometry of a Lunar Eclipse



Now we'll work out how long a lunar eclipse might last for. We will make quite a few simplifying assumptions but even so we can get a fairly good answer. The diagram above is not to scale but gives the idea of the two shadow regions. The Moon in the picture is immersed in what is called the umbra. Notice that in this region, no rays from the Sun should reach the Moon. The lighter shadow region to either side is called the penumbra. In this region, the Earth obscures part of the Sun's disc but some of the Sun's rays reach the Moon's surface.

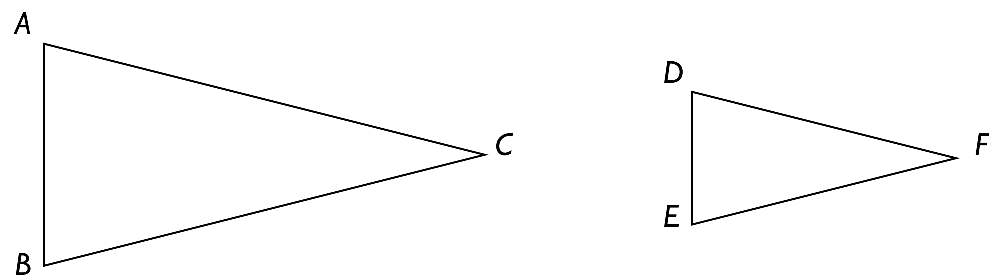
In the photographs of the nice lunar eclipse of March 3 and 4, the Moon is in the umbra at 23.00 and 23.40. If the Earth had no atmosphere it would look completely dark but some of the Sun's rays get refracted (or bent) by the Earth's atmosphere and these illuminate the Moon's surface with a weak red light.

Problem 3: Do you think there is there a connection with the colour of sunsets?

In the photograph at 00.24 a.m. there is a bright illuminated portion (the bright crescent) which has emerged into the penumbra whilst the dark portion is still in the umbra. The exposure times give a good idea of the relative brightness of the Moon in each of the pictures – the longer the exposure time, the fainter was the Moon when the image was made.

We'll need to use similar triangles to help solve this problem. Similar triangles have identical shapes but different sizes or scales. The two triangles below have identical sets of angles and their sides are in a fixed ratio so that

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}.$$

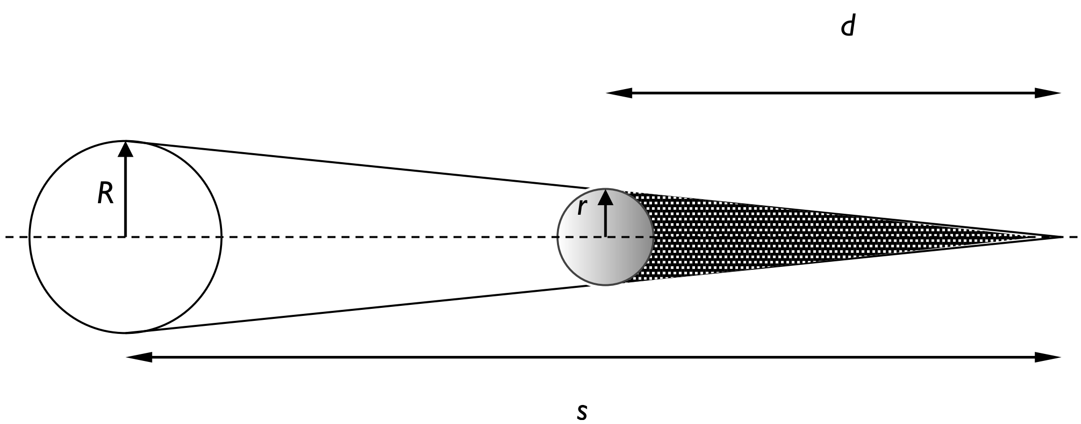


Problem 4: The length of Earth's shadow

To simplify the calculation, we'll break it into parts. The distance from the Earth to the Sun is 149600000 km. From the figure below and using similar triangles, we can see that

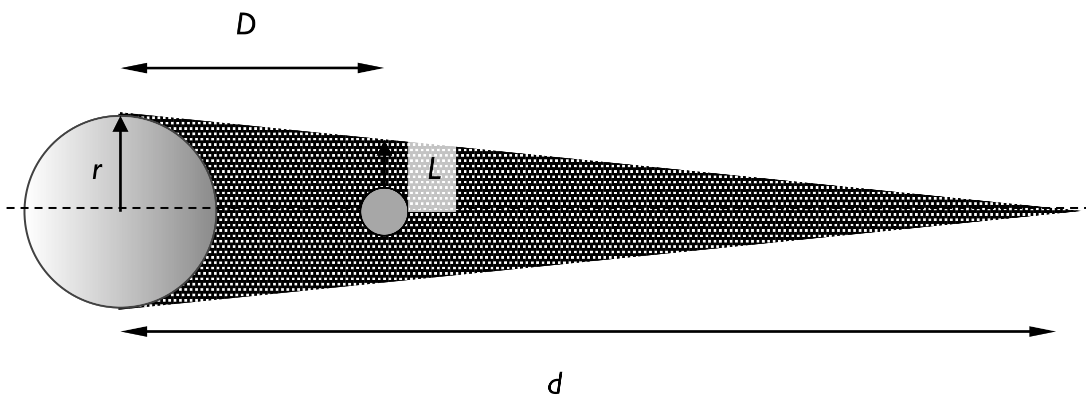
$$\frac{R}{r} = \frac{s}{d} = \frac{d\,\text{km} + 149600000\,\text{km}}{d\,\text{km}}.$$

The radius of the Sun is 695,500 km and the radius of the Earth is 6371 km. Insert these values and so find a value for *d*. This is the furthest distance behind the Earth that we can go and still see the Sun eclipsed by the Earth – the length of the Earth's shadow.



Problem 5: The length of the Moon's track through the shadow

Now we need to figure out the length of the track taken by the Moon through the shadow. For simplicity we approximate the curved orbit of the Moon by a straight line segment as it passes through the shadow.

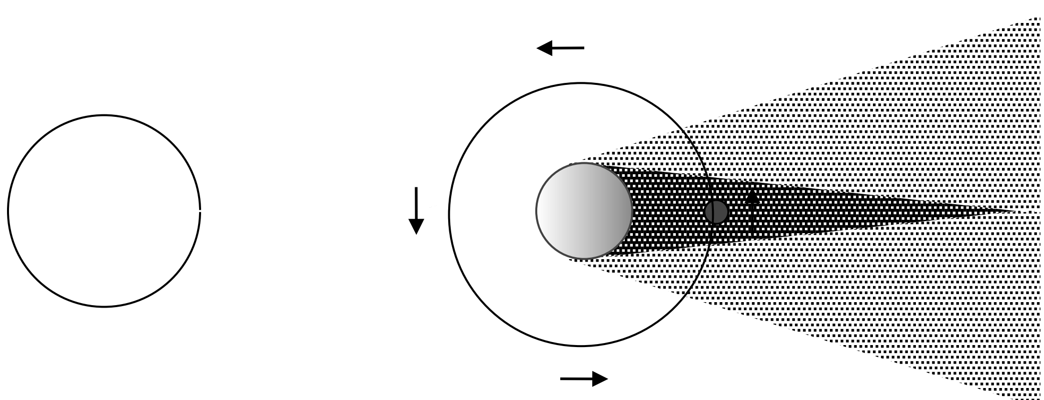


Let *D* be the Earth-Moon distance. The Moon's orbit is not perfectly circular but we'll assume it has a fixed distance of 384400 km. Again looking at the figure and using similar triangles, we can see that, , $\frac{r}{L} = \frac{d}{d-D}$. Put your value for *d* in this equation and hence find the length *L*. The total track length is *2L*.

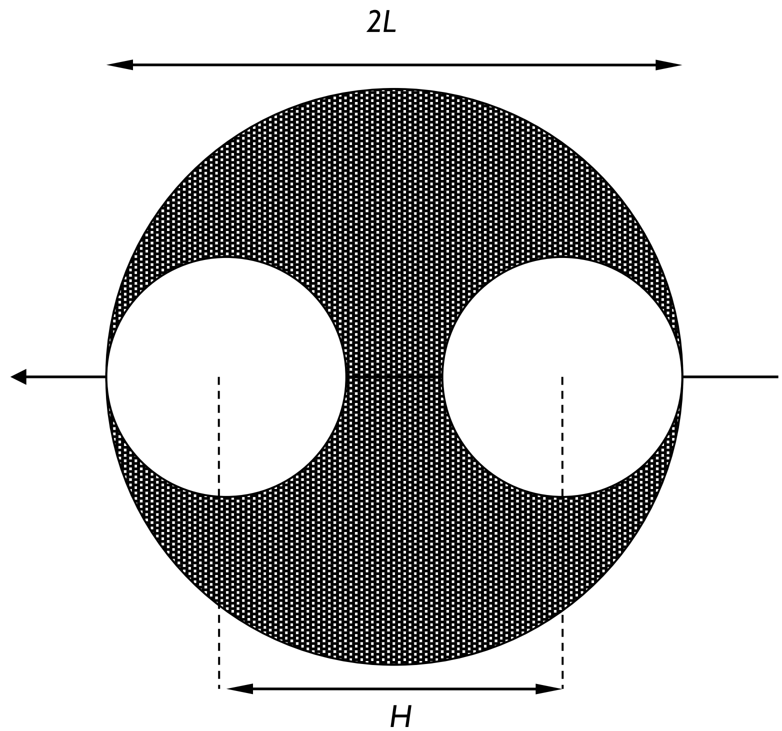
Problem 6: The speed of the Moon in its orbit

With our assumption about the Moon's orbit, we have a total circumference to the orbit of $2\pi D$. The Moon takes 27.321661 days to make one orbit. Convert this to seconds and hence work out the speed of the Moon in its orbit using 'speed = distance/time'.
Now divide the answer to problem 5 by the speed and you get the time it takes for the centre of the Moon to pass from one side of the umbra to the other.

Of course, the shadow will move in space because the Earth is orbiting around the Sun. But let's ignore this for now and assume it is fixed and the Moon moves through a stationary shadow.

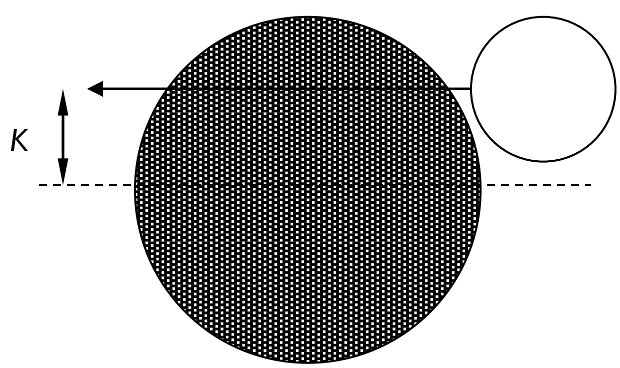


Problem 7: Finding the actual time the disc of the Moon is eclipsed



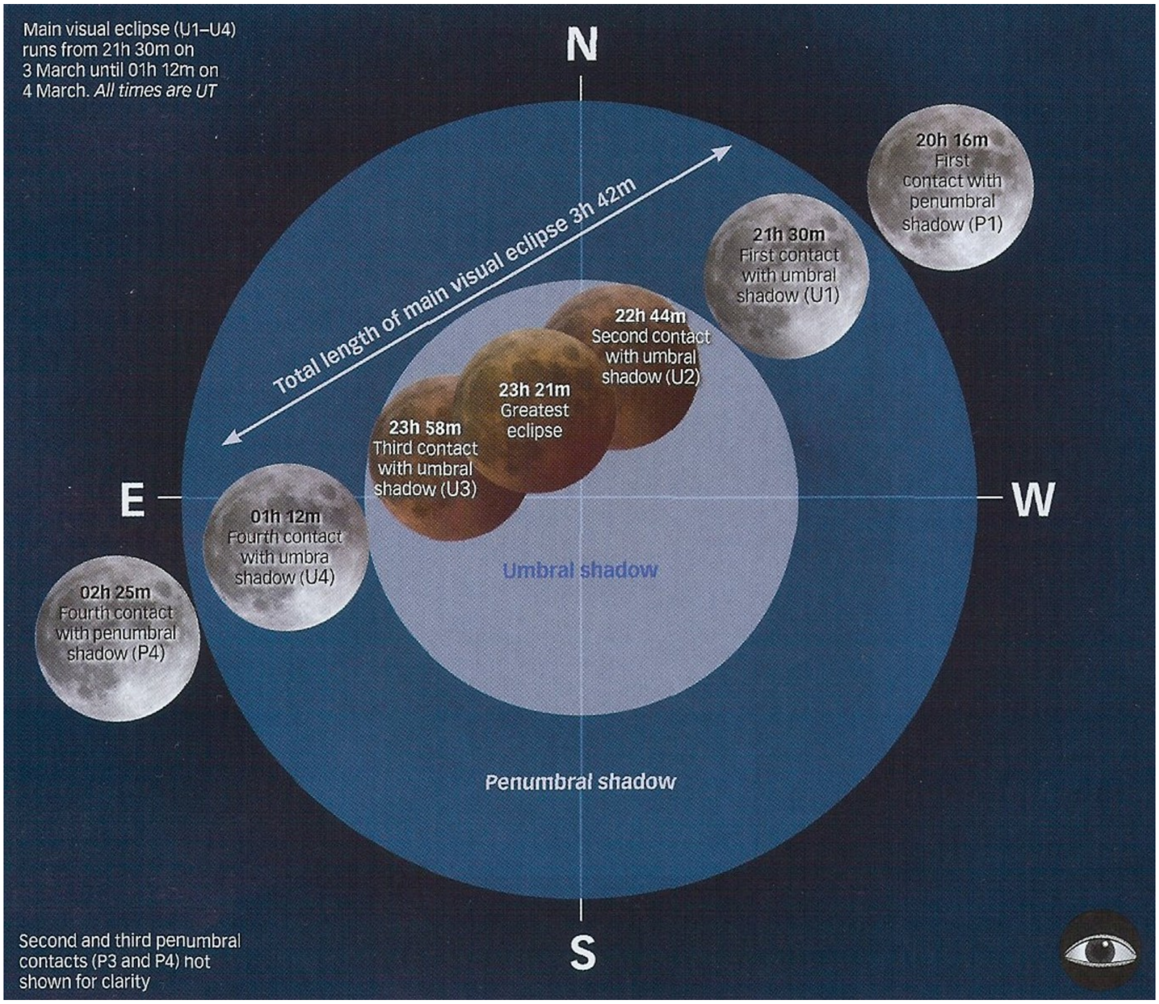
Now we need to correct for the fact the Moon is not a point but has a size. In the picture above, the light discs represent the Moon as it just enters the umbra on the right and just leaves it on the left. So the Moon is only fully eclipsed for the smaller time it takes to move the distance *H* rather than *2L*. The radius of the Moon is 1738 km. So work out the actual time the whole disc is eclipsed.

Problem 8: Will the eclipse be total?



In fact the moon need not pass through the centre of the Earth's shadow and so the eclipse can be shorter.

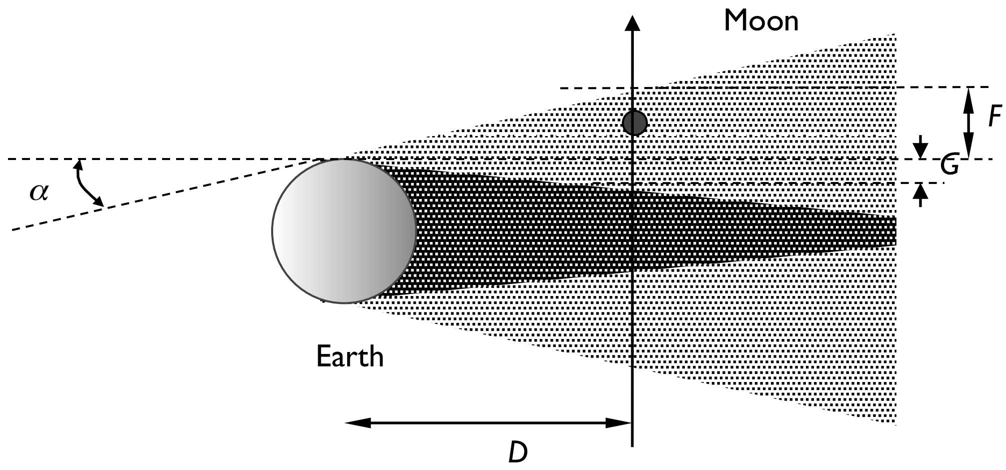
Find how far the track can be from the centre of the umbra and still produce a **total** lunar eclipse.

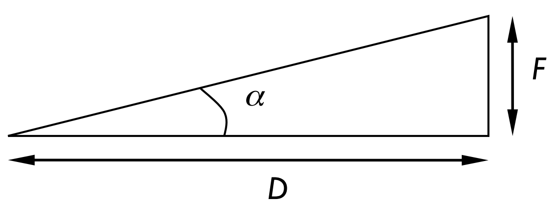


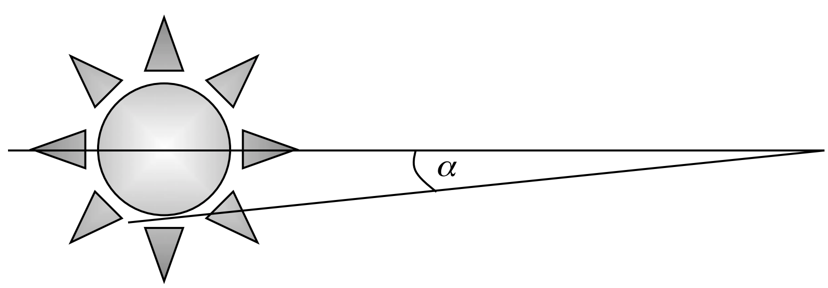
In fact, the Moon did make an off-centre passage through the umbra in the March 3rd eclipse as you can see from the nice schematic above from the UK astronomy magazine Astronomy Now. How close to your answers are the times suggested by the figure?

Problem 9 (if you have time): How long will the Moon spend in the penumbra?

Everything you need to know is contained in the figure below. You need to work out *G* and *F* and then add them up. *G* is just the radius of the Earth 6371 km minus the length *L* you worked out in problem 5. You can use the tan function from problem 1 to work out F. You will need to work out the angle α first though and this is half the angular diameter of the Sun. Given the Sun's radius is 695500 km and its distance is 149600000 km, can you use tan to first find the angle α? The picture should help.


Moon
F
G
α
Earth
D


α
F
D


α

Problem 10: The picture above is a negative of one of the 3rd March lunar eclipse pictures picturing part of the Moon's disc emerging from the umbra. Using compasses fit the best circle you can to the edge of the shadow. Measure the radius of your circle and try and scale it to a real physical size to compare with your calculation. You can use the Moon's size in the picture to do this, remembering that the lunar radius is 1738km.

Problem 11 (if you have time): The picture at left shows an annular eclipse of the Sun when the angular size of the Moon is too small to totally hide the Sun's disc. Why can you never get an annular lunar eclipse? How far away from the Earth would the Moon have to be for this to be possible?